Options for Semi-Lagrangian Trajectory Calculations

Averaging rule: Mid-point/Trapezoidal
Interpolation: Linear/Cubic

Here we compare **mid-point rule** and **trapezoidal rule** for the calculation of displacements $\Delta\mathbf{r}$ in the semi-Lagrangian scheme.

The **mid-point rule** (a time mean followed by a space interpolation) can be described as follows:

$$\Delta\mathbf{r}^{i} = \Delta t \frac{\mathbf{v}(t) + \mathbf{v}(t-\Delta t)}{2}\left(\mathbf{r} - \Delta\mathbf{r}^{i-1}/2\right) = \Delta t \mathbf{v}_{M}$$

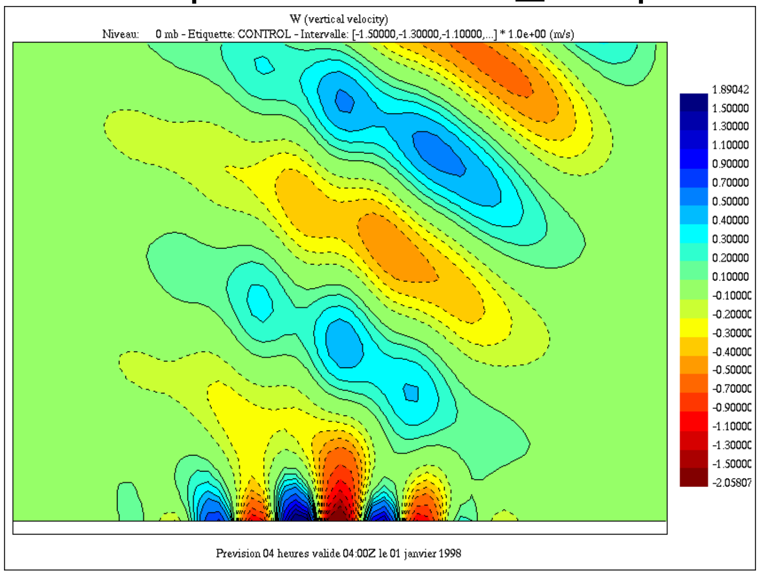
where i is for iterations being made due to the non-linear nature of the process, while the **trapezoidal rule** (a space interpolation followed by a space-time mean) can be written:

$$\Delta\mathbf{r}^{i} = \Delta t \frac{\mathbf{v}(t,\mathbf{r}) + \mathbf{v}\left(t-\Delta t, \mathbf{r} - \Delta\mathbf{r}^{i-1}\right)}{2} = \Delta t \frac{\mathbf{v}_{A} + \mathbf{v}_{D}}{2}$$

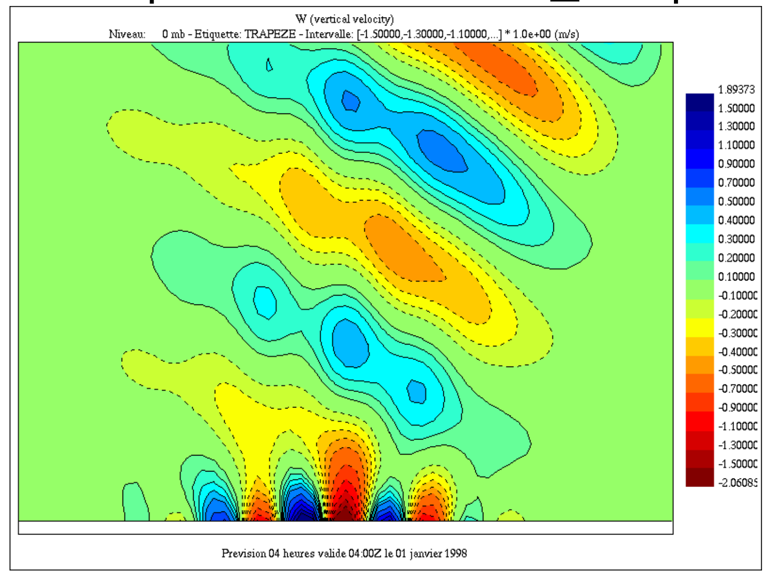
Changing rule is fairly straightforward except for the 'horizontal' on the sphere.

Information: **GEM4.2** Appendix 14**.** Trapezoidal rule for trajectory calculations

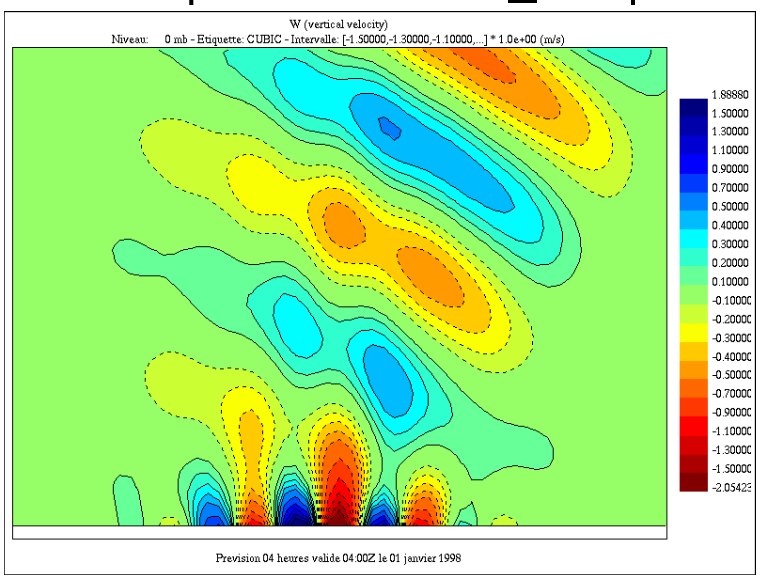
Mid-point rule/linear_interp



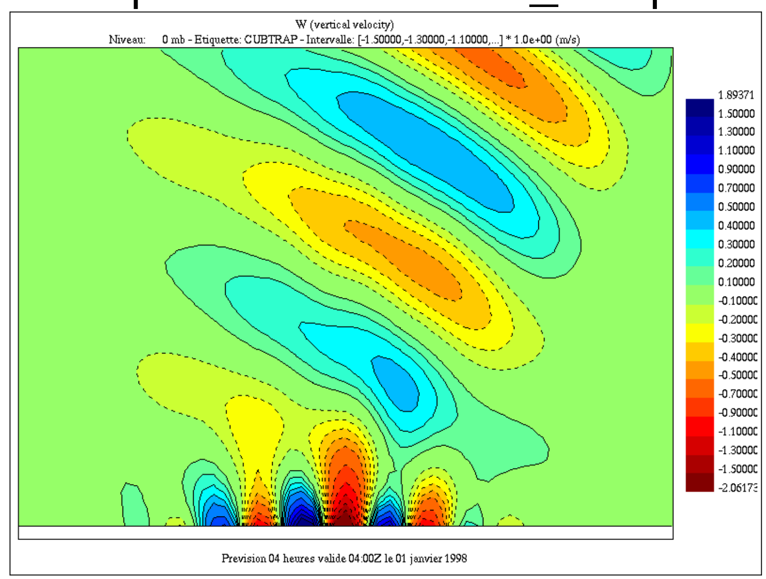
Trapezoïdal rule/linear_interp



Mid-point rule/cubic_interp



Trapezoïdal rule/cubic_interp



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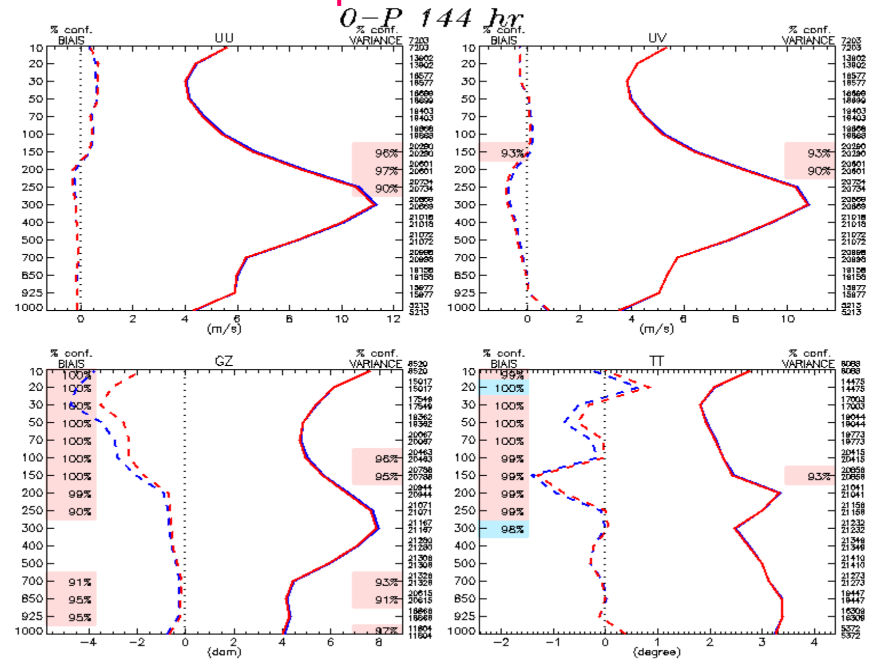
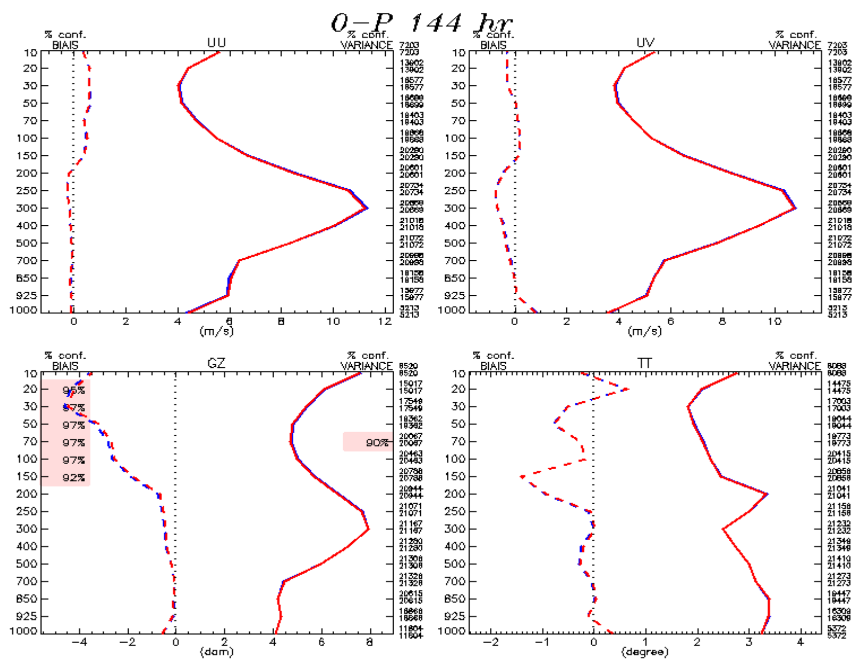
Semi-Lagrangian Trajectory Calculations

Blue: Mid-point rule/linear interpolation
Red: Various modifications

Trapezoidal rule



Cubic interpolation



Trapezoidal rule/cubic interpolation